

**Thermal Model Analysis of Particle Ratios
at GSI Ni–Ni Experiments
Using Exact Strangeness Conservation.**

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Abstract

The production of hadrons in Ni–Ni at the GSI is considered in a hadronic gas model with chemical equilibrium. Special attention is given to the abundance of strange particles which are treated using the exact conservation of strangeness. It is found that all the data can be described using a temperature $T = 70 \pm 10$ MeV and a baryon chemical potential $\mu_B = 720 \pm 20$ MeV.

I. INTRODUCTION

Recent experiments at the GSI laboratory have shown a relatively large yield of K^- mesons [1,2] in collisions involving Ni+Ni at 1.93 GeV A. This result is surprising because the beam energy is below the threshold for the reaction $N + N \rightarrow N + N + K^+ + K^-$ which needs at least 2.5 GeV in the lab. The production of K^- mesons therefore proceeds entirely via medium effects in the nuclear matter of the colliding nuclei. An explanation for this phenomenon has been suggested in several models [3–5] wherein it is argued that the mass of the K^- decreases when nuclear density increases while the mass of the K^+ increases slightly. This would make it easier for K^- mesons to be produced since the threshold for pair production becomes considerably lower. The production of kaons is therefore a test of the effects of the density of the medium and should be analyzed carefully.

It has been pointed out [6,7] that the relative abundance of hadrons (excluding kaons) in the final state can be described by using a hadronic gas model with chemical equilibrium, i.e. only two parameters, the temperature T and the baryon chemical potential μ_B describe all hadronic abundances except the kaons.

It is the purpose of the present paper to address the abundance of kaons in detail. It is well-known that imposing exact strangeness conservation introduces a suppression of strange particles if the system has very small values of the dimensionless quantity VT^3 . For large values of VT^3 this suppression disappears rapidly. The size of the Ni system is relatively large however the temperatures involved are always below 100 MeV and thus these corrections have to be considered seriously. We will show in the present paper that all hadronic ratios, involving strange particles, can be explained with only three parameters, namely the temperature $T \approx 70$ MeV, the baryon chemical potential $\mu_B \approx 720$ MeV and the radius of the system, $R \approx 4$ fm. The large variety of measured production ratios of non-strange particles are explained with the same set of parameter values.

There exists evidence that particle abundances in heavy ion collisions at higher energies are also very close to chemical equilibrium (for a review see e.g. ref. [8]). The data from CERN favor a region around $T \approx 160 - 200$ MeV and a baryon chemical potential around $\mu_B \approx 180 - 350$ MeV. The data from BNL favor a region having a lower temperature, $T \approx 100 - 140$ MeV and a larger chemical potential $\mu_B \approx 450 - 600$ MeV. The results from GSI are in a region which is $T \approx 60 - 100$ MeV and a baryon chemical potential $\mu_B \approx 700$ MeV. The data from GSI thus correspond to a low temperature but a considerably higher value of the baryon chemical potential μ_B . Many questions are still left open. It doesn't explain where the K^- 's are coming from or why they seem to be so "easily" produced despite being sub-threshold. The evidence for chemical equilibrium is very strong in our opinion.

What about thermal equilibrium? Clearly the momentum spectra deviate strongly from a simple thermal distribution given by a Boltzmann factor, however, many of the effects cancel out when considering ratios of hadrons as we do here. Effects due to transverse flow or excluded volume cancel out in ratios (under circumstances described in [9]). In this paper we would like to investigate the production of different particle species as observed at the GSI in Ni–Ni collisions.

II. FORMALISM

To impose strict strangeness conservation one projects the standard (grand canonical) partition function, $Z(T, \lambda_B, \lambda_S, \lambda_Q)$, onto the state with strangeness S

$$Z_S = \frac{1}{2\pi} \int_0^{2\pi} d\phi e^{-iS\phi} Z(T, \lambda_B, \lambda_S, \lambda_Q), \quad (2.1)$$

where the fugacity factor λ_S has been replaced by

$$\lambda_S = e^{i\phi}. \quad (2.2)$$

Since the incoming state has $S = 0$ we will consider only this value in the rest of this analysis. For simplicity we present below the formalism used in Boltzmann approximation. It is more complicated to use quantum ‘statistics’ but it can be done (see e.g. [10–12]). The partition function is then composed of a sum over the different particle species. Those that have strangeness zero are untouched by the above operation, those that have strangeness $+1$ acquire a factor $e^{i\phi}$ while those with strangeness -1 acquire a factor $e^{-i\phi}$. If only strangeness $0, \pm 1$ particles are present we have

$$Z_{S=0} = \frac{1}{2\pi} \int_0^{2\pi} d\phi \exp \{ \mathcal{N}_{S=0} + \mathcal{N}_{S=1} e^{i\phi} + \mathcal{N}_{S=-1} e^{-i\phi} \}, \quad (2.3)$$

where $\mathcal{N}_{S=0}$ is the number of particles having zero strangeness, i.e. mainly pions and nucleons. $\mathcal{N}_{S=\pm 1}$ is defined as the sum over all particles having strangeness ± 1 omitting the fugacity factor due to strangeness, i.e., the contribution of Λ particles to $\mathcal{N}_{S=-1}$ would be given by

$$\mathcal{N}_{S=-1} = V \int \frac{d^3p}{(2\pi)^3} e^{-E_\Lambda/T + \mu_B/T}. \quad (2.4)$$

We have checked that the presence of cascade particles does not modify our final results for the analysis of the GSI data. Of course for higher temperatures cascade particles and Ω ’s should be included. The strange and anti-strange particle numbers are not equal since there is a net baryon number in the system, e.g. there is a difference in the numbers of Λ ’s and $\bar{\Lambda}$ ’s due to the presence of the baryon chemical potential, μ_B .

$$Z_{S=0} = \frac{1}{2\pi} \int_0^{2\pi} d\phi \exp \{ \mathcal{N}_{S=0} + (\mathcal{N}_{S=1} + \mathcal{N}_{S=-1}) \cos \phi + i (\mathcal{N}_{S=1} - \mathcal{N}_{S=-1}) \sin \phi \}. \quad (2.5)$$

Despite its appearance, the above expression has no imaginary part. Exploiting the symmetry properties of the integrand, this can be rewritten as

$$Z_{S=0} = \frac{1}{2\pi} \int_0^{2\pi} d\phi \exp \{ \mathcal{N}_{S=0} + (\mathcal{N}_{S=1} + \mathcal{N}_{S=-1}) \cos \phi \} \cos [(\mathcal{N}_{S=1} - \mathcal{N}_{S=-1}) \sin \phi]. \quad (2.6)$$

The partition function can thus be written in a compact form as

$$Z_{S=0} = Z_0 \frac{1}{2\pi} \int_0^{2\pi} d\phi \cos \{ x \sin \phi \} e^{y \cos \phi}, \quad (2.7)$$

where

$$x = \mathcal{N}_{S=1} - \mathcal{N}_{S=-1} \quad (2.8)$$

and

$$y = \mathcal{N}_{S=1} + \mathcal{N}_{S=-1} \quad (2.9)$$

and Z_0 is the part of the partition function which contains only non-strange particles.

As an example, the number of kaons is given by

$$N_K = gV \int \frac{d^3p}{(2\pi)^3} e^{-E_K/T} \times \frac{\frac{1}{2\pi} \int_0^{2\pi} d\phi \cos\{\phi + x \sin \phi\} e^{y \cos \phi}}{\frac{1}{2\pi} \int_0^{2\pi} d\phi \cos\{x \sin \phi\} e^{y \cos \phi}}. \quad (2.10)$$

It is of interest to investigate the small volume limit of the above expressions since this is where the effects due to the canonical formalism are most obvious. A typical term has the following form

$$Z_{S=0} = \frac{1}{2\pi} \int_0^{2\pi} d\phi \exp \left[\cdots + gV \int \frac{d^3p}{(2\pi)^3} e^{-E/T+i\phi} \cdots \right], \quad (2.11)$$

which in the small volume limit will lead to

$$Z_{S=0} \approx 1 + \frac{1}{2\pi} \int_0^{2\pi} d\phi \left[gV \int \frac{d^3p}{(2\pi)^3} e^{-E/T+i\phi} \right] \times \left[gV \int \frac{d^3p}{(2\pi)^3} e^{-E/T-i\phi} \right] + \cdots. \quad (2.12)$$

Note that each time there will be two terms since otherwise the integration over ϕ leads to a vanishing contribution. The number of K^+ will thus be given by

$$N_{K^+} \approx V \int \frac{d^3p}{(2\pi)^3} e^{-E_{K^+}/T} \left[gV \int \frac{d^3p}{(2\pi)^3} e^{-E_{\bar{K}}/T} + gV \int \frac{d^3p}{(2\pi)^3} e^{-E_{\Lambda}/T+\mu_B/T} \right]. \quad (2.13)$$

This shows clearly that the number of particles increase quadratically with the volume. For a large system the dependence on the volume becomes linear. The additional suppression of strangeness can be seen clearly in the small volume limit above. In order to balance strangeness, the production of a kaon has to be accompanied by either an anti-kaon or by a strange baryon. This is explicitly present in equation 2.13.

The results are not changed substantially if one includes also particles having strangeness ± 2 and ± 3 using the analytical methods of [13,14].

III. NUMERICAL ANALYSIS

A. Resonance Width

As long as the baryon chemical potential is below ≈ 800 MeV the Boltzmann approximation is adequate. For higher values one has to include effects due to quantum statistics. Due

to the low temperatures involved the width of resonances has to be taken into account. For example, an appreciable number of pions is coming from the decay of Δ resonances below the mass of the Δ but still within the width of the Δ . For very small values of the width Γ , the Breit-Wigner resonance shape can be replaced by a δ function:

$$\lim_{\Gamma \rightarrow 0} \frac{1}{\pi} \frac{m\Gamma}{(s - m^2)^2 + m^2\Gamma^2} = \delta(s - m^2). \quad (3.1)$$

For large widths this is no longer possible and one has to keep the Breit-Wigner resonance shape. For example the integration of the Boltzmann factor has to be replaced according to

$$\int d^3p \exp \left[-\frac{\sqrt{p^2 + m^2}}{T} \right] \longrightarrow \int d^3p \int ds \exp \left[-\frac{\sqrt{p^2 + s}}{T} \right] \frac{1}{\pi} \frac{m\Gamma}{(s - m^2)^2 + m^2\Gamma^2}. \quad (3.2)$$

The mass \sqrt{s} is integrated from $m - 2\Gamma$ to $m + 2\Gamma$. In some cases, *e.g.* $\Delta(1232)$, $N(1440)$, the lower limit goes far below the threshold limit, *e.g.* for the Δ resonance $\sqrt{s} = m_N + m_\pi$. In such cases the lower limit is chosen to be the threshold value $m_N + m_\pi$. We have taken care of the fact that the normalization must be adjusted accordingly since the integral over the Breit-Wigner factor should still give unity.

B. Isospin Asymmetry.

In an isospin asymmetric system there are initially four parameters, namely the temperature T , fugacities λ_B and λ_Q , and the volume V . The charge chemical potential can be eliminated by considering the ratio of baryon- and charge content of the system. The baryon- and charge densities, n_B and n_Q , and thus the corresponding chemical potentials, are related by the condition

$$n_B(T, \lambda, R) = 2 \left(\frac{B}{2Q} \right) n_Q(T, \lambda, R), \quad (3.3)$$

where $(\frac{B}{2Q})$ measures the isospin asymmetry in the system. For the Ni-Ni system considered here, one has

$$\left(\frac{B}{2Q} \right)_{\text{Ni-Ni}} = \frac{A_{\text{Ni}}}{2Z_{\text{Ni}}} \simeq 1.04 \quad (3.4)$$

and so there is a 4% deviation from the isospin symmetric case.

C. Results

The hadronic gas model contains all particles listed by the Particle Data Group [15]. Our main result is shown in figure 1. Each experimentally measured hadronic ratio corresponds to a band in the (T, μ_B) plane. The width of the band corresponds to the error bar which has been reported. It is to be noted that several of these bands are almost parallel to the temperature axis. All these bands are fairly insensitive to the chosen hadronic volume. The

main exception is the K^+/π^+ ratio. This ratio is highly sensitive to the value of the radius of the hadronic gas. In figure 1 we show this for a radius of 4 fm. The dependence can be seen more quantitatively in figure 2 where one can see the K^+/π^+ ratio corresponding to a radius of 3.7 fm and also for 4 fm. Also shown is the large volume limit which is denoted by TD (thermodynamic) limit, corresponding to infinite volume. The temperature necessary to reproduce this ratio rapidly goes down with increasing values of the radius. This is because for a small volume there is an intrinsic suppression factor which makes it necessary to go to higher temperature in order to reproduce the same ratio. The region of overlap has a value for the baryon chemical potential of

$$\mu_B = 720 \pm 20 \text{ MeV}. \quad (3.5)$$

It is not so easy to fix the temperature because many of the lines are almost parallel to each other, but a temperature interval given by

$$T = 70 \pm 10 \text{ MeV} \quad (3.6)$$

will give a good fit to the hadronic ratios.

D. The Ratio Λ/K

A requirement of the exact strangeness conservation in an ensemble, where the hadrons i with $|S_i| \leq 1$ are included, leads to condition

$$K^+ + K^0 + (K^*) + \bar{\Lambda} + \bar{\Sigma} + (\bar{Y}^*) = K^- + \bar{K}^0 + (\bar{K}^*) + \Lambda + \Sigma + (Y^*), \quad (3.7)$$

where hadron symbols stand for their densities at freeze-out. Now the baryon chemical potential is quite convincingly fixed to the range $700 \text{ MeV} \leq \mu_B \leq 800 \text{ MeV}$, and the temperature is found to be around $T \sim 70 \text{ MeV}$, so the anti-strange baryons can be neglected due to condition $Y/\bar{Y} \propto \exp(2\mu_B/T) \sim 10^8$. Further, the heavier resonances (denoted by superscript *) are not supposed to play any significant role in energies considered, so

$$\frac{\Lambda + \Sigma}{K^+ + K^0} \simeq 1 - \frac{K^- + \bar{K}^0}{K^+ + K^0}. \quad (3.8)$$

At the first stage, the effect of isospin asymmetry in the system is neglected, and we are led to the result (Σ^0 and Λ are not distinguished)

$$\frac{\Lambda}{K^+} \simeq 1 - \frac{K^-}{K^+} = 0.96 \pm 0.02, \quad (3.9)$$

where the experimental result $K^+/K^- = 26 \pm 9$ is used.

When the charge chemical potential is taken into account, equation (3.8) leads to expression

$$\frac{\Lambda}{K^+} \simeq \frac{(1 + \lambda_Q^{-1}) - (1 + \lambda_Q) \frac{K^-}{K^+}}{1 + \cosh(\ln \lambda_Q)}, \quad (3.10)$$

where $\lambda_Q = \exp(\mu_Q/T)$ is a charge fugacity. Using values $60 \text{ MeV} \leq T \leq 80 \text{ MeV}$ and $700 \text{ MeV} \geq \mu_B \geq 770 \text{ MeV}$ we extract the corresponding values for the charge fugacity ($-2 \text{ MeV} \geq \mu_Q \geq -6 \text{ MeV}$), and obtain the result $\Lambda/K^+ = 0.99 \pm 0.03$. Similar procedure gives for neutral kaons the result $\Lambda/K^0 = 0.94 \pm 0.03$.

IV. SUMMARY

The data from GSI show very good agreement with a hadronic gas in chemical equilibrium having $T \approx 70$ MeV and $\mu_B \approx 720$ MeV. When comparing the GSI results with those obtained at higher energies a clear picture emerges. There is an increase in the temperature and at the same time a decrease in the baryon chemical potential μ_B in going from GSI to Brookhaven (BNL) and then on to CERN. This is indicated schematically in figure 3 where we have made use of the recent compilations of Sollfrank [8] (see also [16]) and of [7]. It is not trivial to translate the values for T and μ_B into values for the energy and the baryon densities. This is because one needs to know the particle numbers and not just the ratios as we have done in this paper. An estimate of these has been made recently in [17]. A good agreement with chemical equilibrium does not mean that the particle spectra should follow exactly a Boltzmann distribution since the momenta of particles can be severely affected by flow. As an example, a model with Bjorken expansion in the longitudinal direction will still have its particle ratios determined by Boltzmann factors even though the longitudinal distribution is nowhere near a Boltzmann distribution [9].

The main deviation from chemical equilibrium is observed in strange particle abundances. These clearly deviate from chemical equilibrium but it seems that a single parameter measuring the deviation from chemical equilibrium is sufficient to describe most of the strange particles. The necessary suppression can be fully accounted for by using exact strangeness conservation and there is no need to introduce a strangeness suppression factor γ_S as is the case for particle abundances measured at higher energies at CERN.

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FIGURES

FIG. 1. Curves in the (μ_B, T) plane corresponding to the hadronic ratios indicated. The interaction volume corresponds to a radius of 4 fm, and the isospin asymmetry is $B/2Q = 1.04$.

FIG. 2. Curves in the (μ_B, T) plane showing the dependence on the radius of the interaction volume. Interaction volume corresponds to radius 4 fm. The K^+/π^+ and ϕ/K^- ratios with the freeze-out radius $R = 3.7$, and the thermodynamic limit for the ratio K^+/π^+ are presented with discrete labels.

FIG. 3. Location of the freeze-out temperature and baryon chemical potential for different energies.





